MATH 147: GUIDELINES AND PRACTICE PROBLEMS FOR EXAM 2

Topics covered on Exam 2.

(i) Constrained optimization problems using Lagrange multipliers.
(ii) Double integrals via iterated integrals and Fubini's Theorem.
(iii) Double integrals via polar coordinates.
(iv) Improper double integrals.
(v) Transformations of $\mathbb{R}^2$, their Jacobians and inverses, especially linear transformations, the one-to-one property.
(vi) Double integrals using the change of variables formula.

Practice problems.

1. Find the absolute minimum and maximum values of the function $f(x,y) = \frac{1}{2}x^2 + y^2$ on the elliptic disk $D : 0 \leq \frac{1}{2}x^2 + y^2 \leq 1$ by finding critical points on the interior of D and using a Lagrange multiplier on the boundary of D.

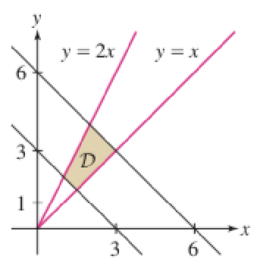
2. Find the maximum and minimum values of $f(x,y,z) = x^2 + y^2 + z^2 - x + y$ subject to the constraint $x^2 + y^2 + z^2 = 1$. Use this to find the absolute maximum and minimum values of $f(x,y,z)$ on the solid $0 \leq x^2 + y^2 + z^2 \leq 1$. Hint: The critical points on the interior of D satisfy $f_x = f_y = f_z = 0$.

3. Find the extreme values of $f(x,y,z) = x + y + z$ subject to the constraints $x^2 + y^2 = 2$ and $x + z - 1 = 0$.

4. Find the maximum and minimum values of $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraints $z^2 = x^2 + y^2$ and $z = x + y - z + 1 = 0$. (Try this on your own, and then if you get stuck, see Example 4.45 in our OS textbook.)

5. OS Chapter 5: # 101,105 (with $\frac{\pi}{2} \leq y \leq \frac{3\pi}{2}$), 115, 147, 159, 163, 178, 365, 389, 391, 393, 395, 398, 431.

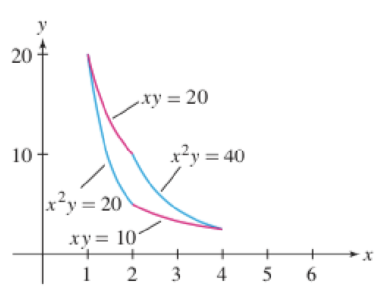
6. Calculate $\int\int_D (x+y)\, dA$, for D



,

using the transformation $G(u,v) = (\frac{u}{v+1}, \frac{uv}{v+1})$.

7. Calculate $\int\int_D e^{xy}\, dA$, for D the region



,

by using the inverse of the transformation $F(x,y) = (xy, x^2y)$. Explain carefully how you obtain the domain of integration in the uv-plane

8. $\int\int_D \sqrt{x+y}(x-y)^2\ dA$, where D is the region bounded by the lines $x=0, y=0.x+y=1$.

9. $\int\int_D \frac{1}{(x^2+y^2)^{\frac{3}{4}}}\ dA$, where D is the disk centered at the origin in $\mathbb{R}^2$ with radius R.

10. $\int\int_{\mathbb{R}^2} e^{-x^2-y^2}\ dA$.

11. $\int\int_D \frac{1}{x^2y^2}\ dA$, where D is the set of points in $\mathbb{R}^2$ satisfying $2 \leq x \leq \infty$ and $2 \leq y \leq \infty$.

12. Compare your answer in problem 11 with $(\int_2^\infty \frac{1}{x^2}\ dx)^2$. Can you explain the relation between these two answers?